Prince Henry's Grammar School
COLLABORATIVE LEARNING TRUST

A Level Mathematics Summer Work 2022

Name: ______________________

Example 2

Expand these expressions and simplify if possible:

a $-3x(7x - 4)$ **b** $y^2(3 - 2y^3)$

c $4x(3x - 2x^2 + 5x^3)$ **d** $2x(5x + 3) - 5(2x + 3)$

a $-3x(7x - 4) = -21x^2 + 12x$

$-3x \times 7x = -21x^{1+1} = -21x^2$
$-3x \times (-4) = +12x$

b $y^2(3 - 2y^3) = 3y^2 - 2y^5$

$y^2 \times (-2y^3) = -2y^{2+3} = -2y^5$

c $4x(3x - 2x^2 + 5x^3)$
$= 12x^2 - 8x^3 + 20x^4$

d $2x(5x + 3) - 5(2x + 3)$
$= 10x^2 + 6x - 10x - 15$
$= 10x^2 - 4x - 15$

Remember a minus sign outside the brackets changes the signs within the brackets.

Simplify $6x - 10x$ to give $-4x$.

Example 3

Simplify these expressions:

a $\dfrac{x^7 + x^4}{x^3}$ **b** $\dfrac{3x^2 - 6x^5}{2x}$ **c** $\dfrac{20x^7 + 15x^3}{5x^2}$

a $\dfrac{x^7 + x^4}{x^3} = \dfrac{x^7}{x^3} + \dfrac{x^4}{x^3}$

Divide each term of the numerator by x^3.

$= x^{7-3} + x^{4-3} = x^4 + x$

x^1 is the same as x.

b $\dfrac{3x^2 - 6x^5}{2x} = \dfrac{3x^2}{2x} - \dfrac{6x^5}{2x}$

Divide each term of the numerator by $2x$.

$= \dfrac{3}{2}x^{2-1} - 3x^{5-1} = \dfrac{3x}{2} - 3x^4$

Simplify each fraction:
$\dfrac{3x^2}{2x} = \dfrac{3}{2} \times \dfrac{x^2}{x} = \dfrac{3}{2} \times x^{2-1}$
$-\dfrac{6x^5}{2x} = -\dfrac{6}{2} \times \dfrac{x^5}{x} = -3 \times x^{5-1}$

c $\dfrac{20x^7 + 15x^3}{5x^2} = \dfrac{20x^7}{5x^2} + \dfrac{15x^3}{5x^2}$
$= 4x^{7-2} + 3x^{3-2} = 4x^5 + 3x$

Divide each term of the numerator by $5x^2$.

Exercise 1A

1 Simplify these expressions:

a $x^3 \times x^4$ **b** $2x^3 \times 3x^2$ **c** $\dfrac{k^3}{k^2}$

d $\dfrac{4p^3}{2p}$ **e** $\dfrac{3x^3}{3x^2}$ **f** $(y^2)^5$

g $10x^5 \div 2x^3$ **h** $(p^3)^2 \div p^4$ **i** $(2a^3)^2 \div 2a^3$

j $8p^4 \div 4p^3$ **k** $2a^4 \times 3a^5$ **l** $\dfrac{21a^3b^7}{7ab^4}$

m $9x^2 \times 3(x^2)^3$ **n** $3x^3 \times 2x^2 \times 4x^6$ **o** $7a^4 \times (3a^4)^2$

p $(4y^3)^3 \div 2y^3$ **q** $2a^3 \div 3a^2 \times 6a^5$ **r** $3a^4 \times 2a^5 \times a^3$

2 Expand and simplify if possible:

a $9(x - 2)$ **b** $x(x + 9)$ **c** $-3y(4 - 3y)$

d $x(y + 5)$ **e** $-x(3x + 5)$ **f** $-5x(4x + 1)$

g $(4x + 5)x$ **h** $-3y(5 - 2y^2)$ **i** $-2x(5x - 4)$

j $(3x - 5)x^2$ **k** $3(x + 2) + (x - 7)$ **l** $5x - 6 - (3x - 2)$

m $4(c + 3d^2) - 3(2c + d^2)$ **n** $(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4)$

o $x(3x^2 - 2x + 5)$ **p** $7y^2(2 - 5y + 3y^2)$ **q** $-2y^2(5 - 7y + 3y^2)$

r $7(x - 2) + 3(x + 4) - 6(x - 2)$ **s** $5x - 3(4 - 2x) + 6$

t $3x^2 - x(3 - 4x) + 7$ **u** $4x(x + 3) - 2x(3x - 7)$ **v** $3x^2(2x + 1) - 5x^2(3x - 4)$

3 Simplify these fractions:

a $\dfrac{6x^4 + 10x^6}{2x}$ **b** $\dfrac{3x^5 - x^7}{x}$ **c** $\dfrac{2x^4 - 4x^2}{4x}$

d $\dfrac{8x^3 + 5x}{2x}$ **e** $\dfrac{7x^7 + 5x^2}{5x}$ **f** $\dfrac{9x^5 - 5x^3}{3x}$

Example 5

Expand these expressions and simplify if possible:

a $x(2x+3)(x-7)$ **b** $x(5x-3y)(2x-y+4)$ **c** $(x-4)(x+3)(x+1)$

a $x(2x+3)(x-7)$

$= (2x^2+3x)(x-7)$

$= 2x^3 - 14x^2 + 3x^2 - 21x$

$= 2x^3 - 11x^2 - 21x$

Start by expanding one pair of brackets: $x(2x+3) = 2x^2 + 3x$

You could also have expanded the second pair of brackets first: $(2x+3)(x-7) = 2x^2 - 11x - 21$ Then multiply by x.

b $x(5x-3y)(2x-y+4)$

$= (5x^2 - 3xy)(2x - y + 4)$

$= 5x^2(2x-y+4) - 3xy(2x-y+4)$

$= 10x^3 - 5x^2y + 20x^2 - 6x^2y + 3xy^2 - 12xy$

$= 10x^3 - 11x^2y + 20x^2 + 3xy^2 - 12xy$

Be careful with minus signs. You need to change every sign in the second pair of brackets when you multiply it out.

c $(x-4)(x+3)(x+1)$

$= (x^2 - x - 12)(x+1)$

$= x^2(x+1) - x(x+1) - 12(x+1)$

$= x^3 + x^2 - x^2 - x - 12x - 12$

$= x^3 - 13x - 12$

Choose one pair of brackets to expand first, for example:

$(x-4)(x+3) = x^2 + 3x - 4x - 12$

$= x^2 - x - 12$

You multiplied together three linear terms, so the final answer contains an x^3 term.

Exercise 1B

1 Expand and simplify if possible:

a $(x+4)(x+7)$ **b** $(x-3)(x+2)$ **c** $(x-2)^2$

d $(x-y)(2x+3)$ **e** $(x+3y)(4x-y)$ **f** $(2x-4y)(3x+y)$

g $(2x-3)(x-4)$ **h** $(3x+2y)^2$ **i** $(2x+8y)(2x+3)$

j $(x+5)(2x+3y-5)$ **k** $(x-1)(3x-4y-5)$ **l** $(x-4y)(2x+y+5)$

m $(x+2y-1)(x+3)$ **n** $(2x+2y+3)(x+6)$ **o** $(4-y)(4y-x+3)$

p $(4y+5)(3x-y+2)$ **q** $(5y-2x+3)(x-4)$ **r** $(4y-x-2)(5-y)$

2 Expand and simplify if possible:

a $5(x+1)(x-4)$ **b** $7(x-2)(2x+5)$ **c** $3(x-3)(x-3)$

d $x(x-y)(x+y)$ **e** $x(2x+y)(3x+4)$ **f** $y(x-5)(x+1)$

g $y(3x-2y)(4x+2)$ **h** $y(7-x)(2x-5)$ **i** $x(2x+y)(5x-2)$

j $x(x+2)(x+3y-4)$

k $y(2x + y - 1)(x + 5)$ **l** $y(3x + 2y - 3)(2x + 1)$

m $x(2x + 3)(x + y - 5)$ **n** $2x(3x - 1)(4x - y - 3)$ **o** $3x(x - 2y)(2x + 3y + 5)$

p $(x + 3)(x + 2)(x + 1)$ **q** $(x + 2)(x - 4)(x + 3)$ **r** $(x + 3)(x - 1)(x - 5)$

s $(x - 5)(x - 4)(x - 3)$ **t** $(2x + 1)(x - 2)(x + 1)$ **u** $(2x + 3)(3x - 1)(x + 2)$

v $(3x - 2)(2x + 1)(3x - 2)$ **w** $(x + y)(x - y)(x - 1)$ **x** $(2x - 3y)^3$

Example 6

Factorise these expressions completely:

a $3x + 9$ **b** $x^2 - 5x$ **c** $8x^2 + 20x$ **d** $9x^2y + 15xy^2$ **e** $3x^2 - 9xy$

a $3x + 9 = 3(x + 3)$ — 3 is a common factor of $3x$ and 9.

b $x^2 - 5x = x(x - 5)$ — x is a common factor of x^2 and $-5x$.

c $8x^2 + 20x = 4x(2x + 5)$ — 4 and x are common factors of $8x^2$ and $20x$. So take $4x$ outside the brackets.

d $9x^2y + 15xy^2 = 3xy(3x + 5y)$ — 3, x and y are common factors of $9x^2y$ and $15xy^2$. So take $3xy$ outside the brackets.

e $3x^2 - 9xy = 3x(x - 3y)$ — x and $-3y$ have no common factors so this expression is completely factorised.

Example 7

Factorise:

a $x^2 - 5x - 6$ **b** $x^2 + 6x + 8$ **c** $6x^2 - 11x - 10$ **d** $x^2 - 25$ **e** $4x^2 - 9y^2$

a $x^2 - 5x - 6$

$ac = -6$ and $b = -5$

So $x^2 - 5x - 6 = x^2 + x - 6x - 6$

$= x(x + 1) - 6(x + 1)$

$= (x + 1)(x - 6)$

Here $a = 1$, $b = -5$ and $c = -6$.

1. Work out the two factors of $ac = -6$ which add to give you $b = -5$. $-6 + 1 = -5$
2. Rewrite the b term using these two factors.
3. Factorise first two terms and last two terms.
4. $x + 1$ is a factor of both terms, so take that outside the brackets. This is now completely factorised.

b $x^2 + 6x + 8$

$= x^2 + 2x + 4x + 8$

$= x(x + 2) + 4(x + 2)$

$= (x + 2)(x + 4)$

$ac = 8$ and $2 + 4 = 6 = b$.

Factorise.

c $6x^2 - 11x - 10$

$= 6x^2 - 15x + 4x - 10$

$= 3x(2x - 5) + 2(2x - 5)$

$= (2x - 5)(3x + 2)$

$ac = -60$ and $4 - 15 = -11 = b$.

Factorise.

d $x^2 - 25$

$= x^2 - 5^2$

$= (x + 5)(x - 5)$

This is the difference of two squares as the two terms are x^2 and 5^2.

The two x terms, $5x$ and $-5x$, cancel each other out.

e $4x^2 - 9y^2$

$= 2^2x^2 - 3^2y^2$

$= (2x + 3y)(2x - 3y)$

This is the same as $(2x)^2 - (3y)^2$.

Example 8

Factorise completely:

a $x^3 - 2x^2$ **b** $x^3 - 25x$ **c** $x^3 + 3x^2 - 10x$

a $x^3 - 2x^2 = x^2(x - 2)$

You can't factorise this any further.

b $x^3 - 25x = x(x^2 - 25)$

$= x(x^2 - 5^2)$

$= x(x + 5)(x - 5)$

x is a common factor of x^3 and $-25x$. So take x outside the brackets.

$x^2 - 25$ is the difference of two squares.

c $x^3 + 3x^2 - 10x = x(x^2 + 3x - 10)$

$= x(x + 5)(x - 2)$

Write the expression as a product of x and a quadratic factor.

Factorise the quadratic to get three linear factors.

Exercise 1C

1 Factorise these expressions completely:

a $4x + 8$
b $6x - 24$
c $20x + 15$
d $2x^2 + 4$
e $4x^2 + 20$
f $6x^2 - 18x$
g $x^2 - 7x$
h $2x^2 + 4x$
i $3x^2 - x$
j $6x^2 - 2x$
k $10y^2 - 5y$
l $35x^2 - 28x$
m $x^2 + 2x$
n $3y^2 + 2y$
o $4x^2 + 12x$
p $5y^2 - 20y$
q $9xy^2 + 12x^2y$
r $6ab - 2ab^2$
s $5x^2 - 25xy$
t $12x^2y + 8xy^2$
u $15y - 20yz^2$
v $12x^2 - 30$
w $xy^2 - x^2y$
x $12y^2 - 4yx$

2 Factorise:

a $x^2 + 4x$
b $2x^2 + 6x$
c $x^2 + 11x + 24$
d $x^2 + 8x + 12$
e $x^2 + 3x - 40$
f $x^2 - 8x + 12$
g $x^2 + 5x + 6$
h $x^2 - 2x - 24$
i $x^2 - 3x - 10$
j $x^2 + x - 20$
k $2x^2 + 5x + 2$
l $3x^2 + 10x - 8$
m $5x^2 - 16x + 3$
n $6x^2 - 8x - 8$
o $2x^2 + 7x - 15$
p $2x^4 + 14x^2 + 24$
q $x^2 - 4$
r $x^2 - 49$
s $4x^2 - 25$
t $9x^2 - 25y^2$
u $36x^2 - 4$
v $2x^2 - 50$
w $6x^2 - 10x + 4$
x $15x^2 + 42x - 9$

3 Factorise completely:

a $x^3 + 2x$
b $x^3 - x^2 + x$
c $x^3 - 5x$
d $x^3 - 9x$
e $x^3 - x^2 - 12x$
f $x^3 + 11x^2 + 30x$
g $x^3 - 7x^2 + 6x$
h $x^3 - 64x$
i $2x^3 - 5x^2 - 3x$
j $2x^3 + 13x^2 + 15x$
k $x^3 - 4x$
l $3x^3 + 27x^2 + 60x$

Example 9

Simplify:

a $\dfrac{x^3}{x^{-3}}$ **b** $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ **c** $(x^3)^{\frac{2}{3}}$ **d** $2x^{1.5} \div 4x^{-0.25}$ **e** $\sqrt[3]{125x^6}$ **f** $\dfrac{2x^2 - x}{x^5}$

a $\dfrac{x^3}{x^{-3}} = x^{3-(-3)} = x^6$

Use the rule $a^m \div a^n = a^{m-n}$.

b $x^{\frac{1}{2}} \times x^{\frac{3}{2}} = x^{\frac{1}{2}+\frac{3}{2}} = x^2$

This could also be written as $\sqrt{x}$.
Use the rule $a^m \times a^n = a^{m+n}$.

c $(x^3)^{\frac{2}{3}} = x^{3 \times \frac{2}{3}} = x^2$

Use the rule $(a^m)^n = a^{mn}$.

d $2x^{1.5} \div 4x^{-0.25} = \frac{1}{2}x^{1.5-(-0.25)} = \frac{1}{2}x^{1.75}$

Use the rule $a^m \div a^n = a^{m-n}$.
$1.5 - (-0.25) = 1.75$

e $\sqrt[3]{125x^6} = (125x^6)^{\frac{1}{3}}$

$= (125)^{\frac{1}{3}}(x^6)^{\frac{1}{3}} = \sqrt[3]{125}(x^{6 \times \frac{1}{3}}) = 5x^2$

Using $a^{\frac{1}{m}} = \sqrt[m]{a}$.

f $\dfrac{2x^2 - x}{x^5} = \dfrac{2x^2}{x^5} - \dfrac{x}{x^5}$

$= 2 \times x^{2-5} - x^{1-5} = 2x^{-3} - x^{-4}$

Divide each term of the numerator by x^5.

$= \dfrac{2}{x^3} - \dfrac{1}{x^4}$

Using $a^{-m} = \dfrac{1}{a^m}$

Example 10

Evaluate:

a $9^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $25^{-\frac{3}{2}}$

a $9^{\frac{1}{2}} = \sqrt{9} = 3$

Using $a^{\frac{1}{m}} = \sqrt[m]{a}$. $9^{\frac{1}{2}} = \sqrt{9}$

b $64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

This means the cube root of 64.

c $49^{\frac{3}{2}} = (\sqrt{49})^3$

$= 7^3 = 343$

Using $a^{\frac{n}{m}} = \sqrt[m]{a^n}$.
This means the square root of 49, cubed.

d $25^{-\frac{3}{2}} = \dfrac{1}{25^{\frac{3}{2}}} = \dfrac{1}{(\sqrt{25})^3}$

$= \dfrac{1}{5^3} = \dfrac{1}{125}$

Using $a^{-m} = \dfrac{1}{a^m}$

Example 11

Given that $y = \frac{1}{16}x^2$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{2}}$ **b** $4y^{-1}$

a $y^{\frac{1}{2}} = \left(\frac{1}{16}x^2\right)^{\frac{1}{2}}$

$= \frac{1}{\sqrt{16}}x^{2 \times \frac{1}{2}} = \frac{x}{4}$

b $4y^{-1} = 4\left(\frac{1}{16}x^2\right)^{-1}$

$= 4\left(\frac{1}{16}\right)^{-1}x^{2 \times (-1)} = 4 \times 16x^{-2}$

$= 64x^{-2}$

Substitute $y = \frac{1}{16}x^2$ into $y^{\frac{1}{2}}$.

$\left(\frac{1}{16}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{16}}$ and $(x^2)^{\frac{1}{2}} = x^{2 \times \frac{1}{2}}$

$\left(\frac{1}{16}\right)^{-1} = 16$ and $x^{2 \times -1} = x^{-2}$

Exercise 1D

1 Simplify:

a $x^3 \div x^{-2}$ **b** $x^5 \div x^7$ **c** $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

d $(x^2)^{\frac{3}{2}}$ **e** $(x^3)^{\frac{5}{3}}$ **f** $3x^{0.5} \times 4x^{-0.5}$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$ **h** $5x^{\frac{7}{5}} \div x^{\frac{2}{5}}$ **i** $3x^4 \times 2x^{-5}$

j $\sqrt{x} \times \sqrt[3]{x}$ **k** $(\sqrt{x})^3 \times (\sqrt[3]{x})^4$ **l** $\frac{(\sqrt[3]{x})^2}{\sqrt{x}}$

2 Evaluate:

a $25^{\frac{1}{2}}$ **b** $81^{\frac{3}{2}}$ **c** $27^{\frac{1}{3}}$

d 4^{-2} **e** $9^{-\frac{1}{2}}$ **f** $(-5)^{-3}$

g $\left(\frac{3}{4}\right)^0$ **h** $1296^{\frac{3}{4}}$ **i** $\left(\frac{25}{16}\right)^{\frac{3}{2}}$

j $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ **k** $\left(\frac{6}{5}\right)^{-1}$ **l** $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$

3 Simplify:

a $(64x^{10})^{\frac{1}{2}}$ **b** $\frac{5x^3 - 2x^2}{x^5}$ **c** $(125x^{12})^{\frac{1}{3}}$ **d** $\frac{x + 4x^3}{x^3}$

e $\frac{2x + x^2}{x^4}$ **f** $\left(\frac{4}{9}x^4\right)^{\frac{3}{2}}$ **g** $\frac{9x^2 - 15x^5}{3x^3}$ **h** $\frac{5x + 3x^2}{15x^3}$

Example 12

Simplify:

a $\sqrt{12}$ **b** $\dfrac{\sqrt{20}}{2}$ **c** $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

a $\sqrt{12} = \sqrt{(4 \times 3)}$

$= \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Look for a factor of 12 that is a square number. Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$. $\sqrt{4} = 2$

b $\dfrac{\sqrt{20}}{2} = \dfrac{\sqrt{4} \times \sqrt{5}}{2}$

$\sqrt{20} = \sqrt{4} \times \sqrt{5}$

$\sqrt{4} = 2$

$= \dfrac{2 \times \sqrt{5}}{2} = \sqrt{5}$

Cancel by 2.

c $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

$= 5\sqrt{6} - 2\sqrt{6}\sqrt{4} + \sqrt{6} \times \sqrt{49}$

$\sqrt{6}$ is a common factor.

$= \sqrt{6}(5 - 2\sqrt{4} + \sqrt{49})$

Work out the square roots $\sqrt{4}$ and $\sqrt{49}$.

$= \sqrt{6}(5 - 2 \times 2 + 7)$

$5 - 4 + 7 = 8$

$= \sqrt{6}(8)$

$= 8\sqrt{6}$

Example 13

Expand and simplify if possible:

a $\sqrt{2}(5 - \sqrt{3})$ **b** $(2 - \sqrt{3})(5 + \sqrt{3})$

a $\sqrt{2}(5 - \sqrt{3})$
$= 5\sqrt{2} - \sqrt{2}\sqrt{3}$
$= 5\sqrt{2} - \sqrt{6}$

$\sqrt{2} \times 5 - \sqrt{2} \times \sqrt{3}$

Using $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

b $(2 - \sqrt{3})(5 + \sqrt{3})$
$= 2(5 + \sqrt{3}) - \sqrt{3}(5 + \sqrt{3})$
$= 10 + 2\sqrt{3} - 5\sqrt{3} - \sqrt{9}$
$= 7 - 3\sqrt{3}$

Expand the brackets completely before you simplify.

Collect like terms: $2\sqrt{3} - 5\sqrt{3} = -3\sqrt{3}$

Simplify any roots if possible: $\sqrt{9} = 3$

Exercise 1E

1 Do not use your calculator for this exercise. Simplify:

a $\sqrt{28}$ **b** $\sqrt{72}$ **c** $\sqrt{50}$

d $\sqrt{32}$ **e** $\sqrt{90}$ **f** $\frac{\sqrt{12}}{2}$

g $\frac{\sqrt{27}}{3}$ **h** $\sqrt{20} + \sqrt{80}$ **i** $\sqrt{200} + \sqrt{18} - \sqrt{72}$

j $\sqrt{175} + \sqrt{63} + 2\sqrt{28}$ **k** $\sqrt{28} - 2\sqrt{63} + \sqrt{7}$ **l** $\sqrt{80} - 2\sqrt{20} + 3\sqrt{45}$

m $3\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}$ **n** $\frac{\sqrt{44}}{\sqrt{11}}$ **o** $\sqrt{12} + 3\sqrt{48} + \sqrt{75}$

2 Expand and simplify if possible:

a $\sqrt{3}(2 + \sqrt{3})$ **b** $\sqrt{5}(3 - \sqrt{3})$ **c** $\sqrt{2}(4 - \sqrt{5})$

d $(2 - \sqrt{2})(3 + \sqrt{5})$ **e** $(2 - \sqrt{3})(3 - \sqrt{7})$ **f** $(4 + \sqrt{5})(2 + \sqrt{5})$

g $(5 - \sqrt{3})(1 - \sqrt{3})$ **h** $(4 + \sqrt{3})(2 - \sqrt{3})$ **i** $(7 - \sqrt{11})(2 + \sqrt{11})$

Example 14

Rationalise the denominator of:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{3 + \sqrt{2}}$ **c** $\frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$ **d** $\frac{1}{(1 - \sqrt{3})^2}$

a $\dfrac{1}{\sqrt{3}} = \dfrac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

Multiply the numerator and denominator by $\sqrt{3}$.

$= \dfrac{\sqrt{3}}{3}$

$\sqrt{3} \times \sqrt{3} = (\sqrt{3})^2 = 3$

b $\dfrac{1}{3 + \sqrt{2}} = \dfrac{1 \times (3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})}$

Multiply numerator and denominator by $(3 - \sqrt{2})$.

$= \dfrac{3 - \sqrt{2}}{9 - 3\sqrt{2} + 3\sqrt{2} - 2}$

$\sqrt{2} \times \sqrt{2} = 2$

$= \dfrac{3 - \sqrt{2}}{7}$

$9 - 2 = 7, -3\sqrt{2} + 3\sqrt{2} = 0$

c $\dfrac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \dfrac{(\sqrt{5} + \sqrt{2})(\sqrt{5} + \sqrt{2})}{(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})}$

Multiply numerator and denominator by $\sqrt{5} + \sqrt{2}$.

$= \dfrac{5 + \sqrt{5}\sqrt{2} + \sqrt{2}\sqrt{5} + 2}{5 - 2}$

$-\sqrt{2}\sqrt{5}$ and $\sqrt{5}\sqrt{2}$ cancel each other out.

$= \dfrac{7 + 2\sqrt{10}}{3}$

$\sqrt{5}\sqrt{2} = \sqrt{10}$

d $\dfrac{1}{(1 - \sqrt{3})^2} = \dfrac{1}{(1 - \sqrt{3})(1 - \sqrt{3})}$

Expand the brackets.

$= \dfrac{1}{1 - \sqrt{3} - \sqrt{3} + \sqrt{9}}$

Simplify and collect like terms. $\sqrt{9} = 3$

$= \dfrac{1}{4 - 2\sqrt{3}}$

$= \dfrac{1 \times (4 + 2\sqrt{3})}{(4 - 2\sqrt{3})(4 + 2\sqrt{3})}$

Multiply the numerator and denominator by $4 + 2\sqrt{3}$.

$\sqrt{3} \times \sqrt{3} = 3$

$= \dfrac{4 + 2\sqrt{3}}{16 + 8\sqrt{3} - 8\sqrt{3} - 12}$

$16 - 12 = 4, 8\sqrt{3} - 8\sqrt{3} = 0$

$= \dfrac{4 + 2\sqrt{3}}{4} = \dfrac{2 + \sqrt{3}}{2}$

Exercise 1F

1 Simplify:

a $\dfrac{1}{\sqrt{5}}$ **b** $\dfrac{1}{\sqrt{11}}$ **c** $\dfrac{1}{\sqrt{2}}$ **d** $\dfrac{\sqrt{3}}{\sqrt{15}}$

e $\dfrac{\sqrt{12}}{\sqrt{48}}$ **f** $\dfrac{\sqrt{5}}{\sqrt{80}}$ **g** $\dfrac{\sqrt{12}}{\sqrt{156}}$ **h** $\dfrac{\sqrt{7}}{\sqrt{63}}$

2 Rationalise the denominators and simplify:

a $\dfrac{1}{1+\sqrt{3}}$ **b** $\dfrac{1}{2+\sqrt{5}}$ **c** $\dfrac{1}{3-\sqrt{7}}$ **d** $\dfrac{4}{3-\sqrt{5}}$ **e** $\dfrac{1}{\sqrt{5}-\sqrt{3}}$

f $\dfrac{3-\sqrt{2}}{4-\sqrt{5}}$ **g** $\dfrac{5}{2+\sqrt{5}}$ **h** $\dfrac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}}$ **i** $\dfrac{11}{3+\sqrt{11}}$ **j** $\dfrac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}}$

k $\dfrac{\sqrt{17}-\sqrt{11}}{\sqrt{17}+\sqrt{11}}$ **l** $\dfrac{\sqrt{41}+\sqrt{29}}{\sqrt{41}-\sqrt{29}}$ **m** $\dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

3 Rationalise the denominators and simplify:

a $\dfrac{1}{(3-\sqrt{2})^2}$ **b** $\dfrac{1}{(2+\sqrt{5})^2}$ **c** $\dfrac{4}{(3-\sqrt{2})^2}$

d $\dfrac{3}{(5+\sqrt{2})^2}$ **e** $\dfrac{1}{(5+\sqrt{2})(3-\sqrt{2})}$ **f** $\dfrac{2}{(5-\sqrt{3})(2+\sqrt{3})}$

Example 1

Solve the following equations:

a $x^2 - 2x - 15 = 0$ **b** $x^2 = 9x$

c $6x^2 + 13x - 5 = 0$ **d** $x^2 - 5x + 18 = 2 + 3x$

a $x^2 - 2x - 15 = 0$

$(x + 3)(x - 5) = 0$

Then either $x + 3 = 0 \Rightarrow x = -3$

or $x - 5 = 0 \Rightarrow x = 5$

So $x = -3$ and $x = 5$ are the two solutions of the equation.

b $x^2 = 9x$

$x^2 - 9x = 0$

$x(x - 9) = 0$

Then either $x = 0$

or $x - 9 = 0 \Rightarrow x = 9$

The solutions are $x = 0$ and $x = 9$.

c $6x^2 + 13x - 5 = 0$

$(3x - 1)(2x + 5) = 0$

Then either $3x - 1 = 0 \Rightarrow x = \frac{1}{3}$

or $2x + 5 = 0 \Rightarrow x = -\frac{5}{2}$

The solutions are $x = \frac{1}{3}$ and $x = -\frac{5}{2}$

d $x^2 - 5x + 18 = 2 + 3x$

$x^2 - 8x + 16 = 0$

$(x - 4)(x - 4) = 0$

Then either $x - 4 = 0 \Rightarrow x = 4$

or $x - 4 = 0 \Rightarrow x = 4$

$\Rightarrow x = 4$

Exercise 2A

1 Solve the following equations using factorisation:

a $x^2 + 3x + 2 = 0$ **b** $x^2 + 5x + 4 = 0$ **c** $x^2 + 7x + 10 = 0$ **d** $x^2 - x - 6 = 0$

e $x^2 - 8x + 15 = 0$ **f** $x^2 - 9x + 20 = 0$ **g** $x^2 - 5x - 6 = 0$ **h** $x^2 - 4x - 12 = 0$

2 Solve the following equations using factorisation:

a $x^2 = 4x$ **b** $x^2 = 25x$ **c** $3x^2 = 6x$ **d** $5x^2 = 30x$

e $2x^2 + 7x + 3 = 0$ **f** $6x^2 - 7x - 3 = 0$ **g** $6x^2 - 5x - 6 = 0$ **h** $4x^2 - 16x + 15 = 0$

3 Solve the following equations:

a $3x^2 + 5x = 2$ **b** $(2x - 3)^2 = 9$ **c** $(x - 7)^2 = 36$ **d** $2x^2 = 8$ **e** $3x^2 = 5$

f $(x - 3)^2 = 13$ **g** $(3x - 1)^2 = 11$ **h** $5x^2 - 10x^2 = -7 + x + x^2$

i $6x^2 - 7 = 11x$ **j** $4x^2 + 17x = 6x - 2x^2$

Example 3

Solve $3x^2 - 7x - 1 = 0$ by using the formula.

$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-1)}}{2 \times 3}$

$a = 3$, $b = -7$ and $c = -1$.

Put brackets around any negative values.

$x = \frac{7 \pm \sqrt{49 + 12}}{6}$

$-4 \times 3 \times (-1) = +12$

$x = \frac{7 \pm \sqrt{61}}{6}$

Then $x = \frac{7 + \sqrt{61}}{6}$ or $x = \frac{7 - \sqrt{61}}{6}$

Or $x = 2.47$ (3 s.f.) or $x = -0.135$ (3 s.f.)

Exercise 2B

1 Solve the following equations using the quadratic formula.
Give your answers exactly, leaving them in surd form where necessary.

a $x^2 + 3x + 1 = 0$ **b** $x^2 - 3x - 2 = 0$ **c** $x^2 + 6x + 6 = 0$ **d** $x^2 - 5x - 2 = 0$

e $3x^2 + 10x - 2 = 0$ **f** $4x^2 - 4x - 1 = 0$ **g** $4x^2 - 7x = 2$ **h** $11x^2 + 2x - 7 = 0$

2 Solve the following equations using the quadratic formula.
Give your answers to three significant figures.

a $x^2 + 4x + 2 = 0$ **b** $x^2 - 8x + 1 = 0$ **c** $x^2 + 11x - 9 = 0$ **d** $x^2 - 7x - 17 = 0$

e $5x^2 + 9x - 1 = 0$ **f** $2x^2 - 3x - 18 = 0$ **g** $3x^2 + 8 = 16x$ **h** $2x^2 + 11x = 5x^2 - 18$

3 For each of the equations below, choose a suitable method and find all of the solutions.
Where necessary, give your answers to three significant figures.

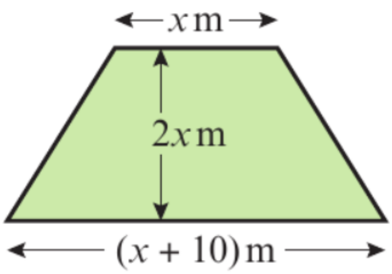
a $x^2 + 8x + 12 = 0$ **b** $x^2 + 9x - 11 = 0$

c $x^2 - 9x - 1 = 0$ **d** $2x^2 + 5x + 2 = 0$

e $(2x + 8)^2 = 100$ **f** $6x^2 + 6 = 12x$

g $2x^2 - 11 = 7x$ **h** $x = \sqrt{8x - 15}$

4 This trapezium has an area of $50\,m^2$.
Show that the height of the trapezium is equal to $5(\sqrt{5} - 1)$ m.



Example 1

Solve the simultaneous equations:

a $2x + 3y = 8$
$3x - y = 23$

b $4x - 5y = 4$
$6x + 2y = 25$

a $2x + 3y = 8$ (1)
$3x - y = 23$ (2)
$9x - 3y = 69$ (3)
$11x = 77$
$x = 7$

$14 + 3y = 8$
$3y = 8 - 14$
$y = -2$

The solution is $x = 7$, $y = -2$.

b $4x - 5y = 4$ (1)
$6x + 2y = 25$ (2)
$12x - 15y = 12$ (3)
$12x + 4y = 50$ (4)
$-19y = -38$
$y = 2$

$4x - 10 = 4$
$4x = 14$
$x = 3\frac{1}{2}$

The solution is $x = 3\frac{1}{2}$, $y = 2$.

Exercise 3A

1 Solve these simultaneous equations by elimination:

a $2x - y = 6$
$4x + 3y = 22$

b $7x + 3y = 16$
$2x + 9y = 29$

c $5x + 2y = 6$
$3x - 10y = 26$

d $2x - y = 12$
$6x + 2y = 21$

e $3x - 2y = -6$
$6x + 3y = 2$

f $3x + 8y = 33$
$6x = 3 + 5y$

2 Solve these simultaneous equations by substitution:

a $x + 3y = 11$
$4x - 7y = 6$

b $4x - 3y = 40$
$2x + y = 5$

c $3x - y = 7$
$10x + 3y = -2$

d $2y = 2x - 3$
$3y = x - 1$

3 Solve these simultaneous equations:

a $3x - 2y + 5 = 0$
$5(x + y) = 6(x + 1)$

b $\dfrac{x - 2y}{3} = 4$
$2x + 3y + 4 = 0$

c $3y = 5(x - 2)$
$3(x - 1) + y + 4 = 0$

4 $3x + ky = 8$
$x - 2ky = 5$
are simultaneous equations where k is a constant.

a Show that $x = 3$.

b Given that $y = \frac{1}{2}$ determine the value of k.

Example 3

Solve the simultaneous equations:

$x + 2y = 3$
$x^2 + 3xy = 10$

$$x + 2y = 3 \quad (1)$$
$$x^2 + 3xy = 10 \quad (2)$$
$$x = 3 - 2y$$
$$(3 - 2y)^2 + 3y(3 - 2y) = 10$$
$$9 - 12y + 4y^2 + 9y - 6y^2 = 10$$
$$-2y^2 - 3y - 1 = 0$$
$$2y^2 + 3y + 1 = 0$$
$$(2y + 1)(y + 1) = 0$$
$$y = -\tfrac{1}{2} \text{ or } y = -1$$
So $x = 4$ or $x = 5$

Solutions are $x = 4$, $y = -\frac{1}{2}$ and $x = 5$, $y = -1$.

Exercise 3B

1 Solve the simultaneous equations:

a $x + y = 11$
$xy = 30$

b $2x + y = 1$
$x^2 + y^2 = 1$

c $y = 3x$
$2y^2 - xy = 15$

d $3a + b = 8$
$3a^2 + b^2 = 28$

e $2u + v = 7$
$uv = 6$

f $3x + 2y = 7$
$x^2 + y = 8$

2 Solve the simultaneous equations:

a $2x + 2y = 7$
$x^2 - 4y^2 = 8$

b $x + y = 9$
$x^2 - 3xy + 2y^2 = 0$

c $5y - 4x = 1$
$x^2 - y^2 + 5x = 41$

3 Solve the simultaneous equations, giving your answers in their simplest surd form:

a $x - y = 6$
$xy = 4$

b $2x + 3y = 13$
$x^2 + y^2 = 78$

4 Solve the simultaneous equations:

$x + y = 3$
$x^2 - 3y = 1$

5 a By eliminating y from the equations

$$y = 2 - 4x$$
$$3x^2 + xy + 11 = 0$$

show that $x^2 - 2x - 11 = 0$.

b Hence, or otherwise, solve the simultaneous equations

$$y = 2 - 4x$$
$$3x^2 + xy + 11 = 0$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.